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5D fuzzball geometries and 4D polar states

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Abstract: We analyze the map between a class of 'fuzzball' solutions in five dimensions and four-dimensional multicentered solutions under the 4D-5D connection, and interpret the resulting configurations in the framework of Denef and Moore [1]. In five dimensions, we consider Kaluza-Klein monopole supertubes with circular profile which represent microstates of a small black ring. The resulting four-dimensional configurations are, in a suitable duality frame, polar states consisting of stacks of D6 and anti-D6 branes with flux. We argue that these four-dimensional configurations represent zero-entropy constituents of a 2-centered configuration where one of the centers is a small black hole. We also discuss how spectral flow transformations in five dimensions, leading to configurations with momentum, give rise to four-dimensional D6 anti-D6 polar configurations with different flux distributions at the centers.

Keywords: Black Holes in String Theory, D-branes, AdS-CFT Correspondence.

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1. Introduction and summary

Recent years have seen a significant progress in the understanding of the supergravity description of BPS states of string theory, both in four and five noncompact dimensions. In four dimensions, it has been established that BPS states of a given charge are often realized as multicentered solutions in supergravity [2 – 4, 1, 5]. An important class of multicentered configurations are the 'polar' states for which no single-centered solution exists and which contribute to the polar part of the OSV partition function [6] regarded as a generalized modular form. From the knowledge of their microscopic degeneracies, the full partition function was reconstructed in [1], leading to a derivation of an OSV-type relation. Another important type of configurations are the so-called 'scaling' solutions, which carry the same charges as a (large) black hole and can be seen as a deconstruction of the black hole into zero-entropy constituents [7].

On the five-dimensional side as well, the BPS objects are not restricted to single-centered black holes. There also exist supersymmetric black rings and black hole-black

ring composites [8 – 10], see [11] for review and a more complete list of references. There are also Kaluza-Klein monopole supertube solutions which carry the charges of a black hole or black ring and are smooth and horizonless [12 – 25]. These can be seen as gravity duals to individual microstates in the CFT description of the black hole, leading to the 'fuzzball' picture proposed by Mathur and collaborators (see [26, 27] for reviews and further references). In this proposal, the black hole horizon is an artefact of an averaging procedure over an ensemble of such smooth solutions.

These zoos of four and five-dimensional BPS configurations are not unrelated, and it is often possible to continuously interpolate between 4D and 5D configurations using the '4D-5D connection' [28 – 33]. Five-dimensional configurations can often be embedded in Taub-NUT space in a supersymmetric manner. The spatial geometry of Taub-NUT space interpolates between $\mathbf{R}^4$ near the origin and $\mathbf{R}^3 \times S^1$ at infinity. By varying the size of the S^1, one can then interpolate between effectively five and four-dimensional configurations. Under this map, a point-like configuration at the center of Taub-NUT space becomes a 4D pointlike solution with added Kaluza-Klein monopole charge. A ring-like configuration at some distance from the center goes over into a two-centered solution where one center comes from the wrapped ring and the other contains Kaluza-Klein monopole charge. Angular momentum in 5D goes over into linear momentum along S^1 in four dimensions.

The goal of the current work is to give an explicit mapping between supertube solutions arising in the fuzzball picture in five dimensions and multi-centered solutions in four dimensions under the 4D-5D connection, and to interpret the resulting configurations using the tools developed in [1]. We will work in toroidally compactified type II string theory, and consider a symmetric class of 2-charge supertubes which are described by a circular profile [12 – 15], as well as 3-charge solutions obtained from those under spectral flow [16 – 19]. Placing such supertubes in Taub-NUT space gives the solutions that were constructed in [20, 22]. Applying the 4D-5D connection, we will show that, in the standard type IIB duality frame, one obtains 4D solutions which are two-centered Kaluza-Klein monopole-antimonopole pairs carrying flux-induced D1 and D5-brane charge and momentum. These solutions can be described within an STU-model truncation of $N = 8$ supergravity and can be seen as simple examples of 'bubbled' solutions [34 – 42] (for a review, see ([43]). To make contact with the techniques developed for analyzing multicentered configurations in Calabi-Yau compactifications, we will transform these configurations to a type IIA duality frame where all charges and dipole moments carried arise from a D6-D4-D2-D0 brane system. In this duality frame, the relevant configurations are two stacks of D6-branes and anti-D6 branes with worldvolume fluxes turned on. Those configurations fall into the class of 'polar' states in 4D for which no single centered solution exists.

Let us briefly summarize our results. We consider 5D supertube solutions carrying D1 charge N_1, D5 charge N_5 and momentum P and which are the gravity duals of a class of symmetric states in the D1-D5 CFT with quantum numbers

$$\begin{aligned} L_0 &= N_1 N_5 \left(m^2 + \frac{m}{n} + 1/4\right), \quad \bar{L}_0 = \frac{N_1 N_5}{4}, \\ J^3 &= -\frac{N_1 N_5}{2}\left(2m + \frac{1}{n}\right), \qquad \bar{J}^3 = -\frac{N_1 N_5}{2n}, \quad P = L_0 - \bar{L}_0 = N_1 N_5 m \left(m + \frac{1}{n}\right). \end{aligned} \tag{1.1}$$

$q_0:$	$D0$	$p^0:$	$D6(T_1\times T_2\times T_3)$
$q_1:$	$D2(T_1)$	$p^1:$	$D4(T_2\times T_3)$
$q_2:$	$D2(T_2)$	$p^2:$	$D4(T_1\times T_3)$
$q_3:$	$D2(T_3)$	$p^3:$	$D4(T_1\times T_2)$

Table 1: Type IIA D-brane charges carried by our configurations. We have denoted the submanifold wrapped by the brane in brackets.

These represent Ramond sector states that are in a right-moving ground state and, on the left-hand side, excited states in a twisted sector. The integer n labels the twist sector and should be a divisor of N_1N_5. In a component string picture, n represents the length of the component strings. These states can be seen as obtained from Ramond ground states through a left-moving spectral flow transformation determined by the parameter m, which should be an integer. They carry momentum only when m is nonzero.

After applying the 4D-5D connection to these configurations, we will interpret them in a U-dual type IIA frame where all the charges arise from D6-D4-D2-D0 branes. Only 4 electric charges q_I and magnetic charges p^I are turned on in these solutions. They arise from wrapping D-branes on the internal cycles given in table 1.

Under the 4D-5D connection, the 5D quantum numbers (1.1) map to the following 4D charges

$$\begin{array}{lcccc} 5D: & N_1 & N_5 & J^3 & \bar{J}^3 & P \\ 4D: & p^2 & p^3 & -\frac{q_1}{2} & -J_z & -q_0 \end{array} \tag{1.2}$$

Writing charges as $\Gamma=(p^I,q_I)$, the 4D BPS state corresponding to (1.1) carries the charge

$$\Gamma_{\rm tot}=\left(0,1,N_1,N_5,\left(2m+\frac{1}{n}\right)N_1N_5,0,0,-m\left(m+\frac{1}{n}\right)N_1N_5\right)\,. \tag{1.3}$$

This is a polar charge for which there is no single-centered solution. It is realized as a two-centered solution consisting of two stacks of D6 and anti-D6 branes with fluxes. Writing the charge as an element of the even cohomology as we will explain in section 2, the charges are

$$\begin{aligned} \Gamma_1 &= -ne^{-\left(m+\frac{1}{n}\right)\omega_1+mN_1\omega_2+mN_5\omega_3}\,, \\ \Gamma_2 &= ne^{-m\omega_1+\left(m+\frac{1}{n}\right)N_1\omega_2+\left(m+\frac{1}{n}\right)N_5\omega_3}\,. \end{aligned} \tag{1.4}$$

The length of the component string n has become the number of D6 and anti-D6 branes in the 4D picture, while the spectral flow parameter m has become a flux parameter. The restrictions on these parameters from charge quantization match the quantization conditions in the CFT description.

This paper is organized as follows. In section 2, we review the construction of multi-centered solutions in the STU-model and construct the solutions with charges (1.4). We explain why these are polar states and review the corresponding split attractor flow trees. In section 3, we transform to a U-dual type IIB duality frame and discuss the lift of our solutions to 10 dimensions. We show that the solutions represent supertubes embedded

in Taub-NUT space, and discuss the 5D limit. In section 4, we discuss the microscopic interpretation of our configurations from the 4D and 5D points of view. We end with some prospects for future research in section 5. In appendix A, we discuss in detail the reduction formulae in the type IIB duality frame.

2. A class of polar states in $N = 8$ supergravity

In this section we construct 2-center solutions in type IIA on a six-torus containing D6 and anti-D6 branes with flux (1.4), and discuss the corresponding split attractor flow. These solutions can be described in a truncation to an STU-model which we presently review.

2.1 STU-truncation of type IIA on T^6

We consider type IIA string theory compactified on a six-torus, which reduces in the low-energy limit to $N = 8$ supergravity in four dimensions. In $N = 2$ language, the $N = 8$ gravity multiplet decomposes into the $N = 2$ gravity multiplet, 6 gravitini multiplets, 15 vector multiplets, and 10 hypermultiplets. For our purposes, it will be sufficient to consider a consistent truncation to a sector where only gravity and 3 vector multiplets are excited. This sector is described by the well-known STU model [44, 45] consisting of $N = 2$ supergravity coupled to 3 vector multiplets with symmetric prepotential

$$F = -D_{ABC}\frac{X^A X^B X^C}{X^0} = -\frac{X^1 X^2 X^3}{X^0}\,,$$

where $D_{ABC} = \frac{1}{6}|\epsilon_{ABC}|$. The bosonic part of the action is given by

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-G}\Big[R - \frac{1}{2}\sum_{A=1}^{3}\frac{\partial_\mu z^A \partial^\mu \bar{z}^A}{(\mathrm{Im}z^A)^2} + \frac{\beta^2}{2}\mathrm{Im}\mathcal{N}_{IJ}\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J\,\mu\nu} + \frac{\beta^2}{4}\mathrm{Re}\mathcal{N}_{IJ}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma}\Big]\,. \quad (2.1)$$

with $z^A = X^A/X^0 \equiv a_A + ib_A$, $A = 1,2,3$, $I = 0,1,2,3$ and $\epsilon^{0123} \equiv 1$. We have left an arbitrary normalization constant β in front of the kinetic terms of the $\mathrm{U}(1)$ fields for easy comparison with different conventions used in the literature. The matrix $\mathcal{N}$ is given by

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2\mathrm{i}\frac{\mathrm{Im}(F_{IK})X^K \mathrm{Im}(F_{JL})X^L}{\mathrm{Im}(F_{MN})X^M X^N}\,. \quad (2.2)$$

where $F_{IJ} = \frac{\partial}{\partial X^I}\frac{\partial}{\partial X^J}F$. The explicit form of $\mathcal{N}$ can be found in the appendix (A.10). In our conventions, the scalars b_A have to be positive in order to have the correct kinetic term for the $\mathrm{U}(1)$ fields.

We will, for simplicity, choose the hypermultiplet moduli such that the six-torus is metrically a product of three 2-tori $T_1 \times T_2 \times T_3$[1]. The 10-dimensional origin of the fields

[1]For later convenience, we also take T_1 to be rectangular and denote its two circles by S^4, S^5.

in (2.1) is the following. The vector multiplet scalars $z^A = X^A/X^0, A = 1,2,3$ describe complexified Kähler deformations of the tori T_A:

$$B + iJ = z^A \omega_A \,, \tag{2.3}$$

where ω_A are normalized volume forms on T_A satisfying $\int_{T_A} \omega_B = \delta^A_B$. The constants D_{ABC} entering in the prepotential are proportional to the intersection numbers: $D_{ABC} = \frac{1}{6}\int \omega_A \wedge \omega_B \wedge \omega_C$. The four U(1) field strengths $\mathcal{F}^I = d\mathcal{A}^I, I = 0,\ldots,3$ arise from dimensional reduction of the RR sector. Charged BPS states can carry electric and magnetic charges under the four U(1) fields. We will denote the magnetic charges by p^I and the electric charges by q_I and write a general charge vector Γ either by a row vector or an element of the even cohomology of T^6:

$$\Gamma = (p^0, p^A, q_A, q_0) = p^0 + p^A\omega_A + q_A\omega^A + q_0\omega_{\rm vol} \,, \tag{2.4}$$

with $\omega^A = 3D_{ABC}\omega_B \wedge \omega_C$ and $\omega_{\rm vol} = \omega_1 \wedge \omega_2 \wedge \omega_3$ and $A = 1,\ldots,3$. Taking into account charge quantization, the components p^I, q_I should be integers or $\Gamma \in H^{\rm even}(T^6, \mathbf{Z})$. We also define a symplectic inner product as

$$\langle \Gamma, \tilde{\Gamma} \rangle = -p^0\tilde{q}_0 + p^A\tilde{q}_A - q_A\tilde{p}^A + q_0\tilde{p}^0 \,. \tag{2.5}$$

From a 10-dimensional point of view, the charged BPS states arise from D-branes wrapping internal cycles. The D-brane interpretation of the charges is given in table 1. Dimensionally reducing the D-brane Born-Infeld and Wess-Zumino action leads to point-particle source terms to be added to the bulk action [46] (2.1):

$$S_{\rm source} = \frac{\beta}{G_4}\int \left[-|Z(Q)|ds + \frac{\beta}{2}\langle Q, \mathcal{A}\rangle \right] . \tag{2.6}$$

Here, Q is a vector whose components have the dimension of length defined as

$$\int_{S^2} \mathcal{F}^I = 4\pi Q^I \qquad \int_{S^2} \mathcal{G}_I = 4\pi Q_I \tag{2.7}$$

Where $\mathcal{G}_I = {\rm Im}\mathcal{N}_{IJ} \star \mathcal{F}^J + {\rm Re}\mathcal{N}_{IJ}\mathcal{F}^J$ and $\star$ denotes the Hodge dual. For later convenience, as we will be taking the size of one of the internal directions to infinity, it will be useful to work in conventions where we do not fix the coordinate volume of the internal cycles. The components of Q are then given by[2]

$$Q^I = \frac{\sqrt{8}}{\beta} T^I V^I G_4 p^I \,, \qquad Q_I = \frac{\sqrt{8}}{\beta} T_I V_I G_4 q_I \,. \tag{2.8}$$

where T^I, T_I are the tensions of the branes in table 1 and the V^I, V_I are the coordinate volumes of the cycles they are wrapping. The quantity $Z(Q)$ in (2.6) is the central charge

$$Z = \langle Q, \Omega \rangle \,, \tag{2.9}$$

[2]To find agreement with [4], one should take the coordinate volume of all cycles equal to one in units of $2\pi\sqrt{\alpha'}$. In that case, the relation between Q and Γ is $Q = \frac{\sqrt{G_4}}{\beta}\Gamma$. Furthermore, $\beta = 1$ in [4].

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and Ω is the normalized period vector defined as

$$\Omega = \frac{\Omega_{\rm hol}}{\sqrt{8b_1b_2b_3}}\,, \tag{2.10}$$

with $\Omega_{\rm hol} = -e^{z^A\omega_A}$. A stack of D-branes with worldvolume flux F turned on sources lower D-brane charges according to

$$\Gamma = \mathrm{Tr}\, e^F\,. \tag{2.11}$$

We will denote this particular embedding of the STU model in toroidally compactified type II string theory as 'duality frame A' in what follows. Later, in section 3, we will also consider an embedding of the STU model into a U-dual type IIB duality frame which we will call 'frame B'.

2.2 Multicentered BPS solutions

We will now review the construction of general multicentered BPS solutions in the STU model considered above, along the lines of Bates and Denef [4]. Such solutions can be constructed from the harmonic functions

$$H^I = h^I + \sum_s \frac{Q^I}{|\vec{x}-\vec{x}_s|}; \qquad H_I = h_I + \sum_s \frac{Q_I}{|\vec{x}-\vec{x}_s|}\,, \tag{2.12}$$

where the index s runs over the centers and x_s are the locations of the centers in $\mathbf{R^3}$.

The metric and gauge fields are then completely determined from the knowledge of a single function $\Sigma(H)$ on $\mathbf{R^3}$:

$$\Sigma(H) = \sqrt{\frac{4x_1x_2x_3 - L^2}{(H^0)^2}}\,, \tag{2.13}$$

with

$$\begin{aligned} x_A &= 3D_{ABC}H^BH^C - H_AH^0\,, \\ L &= 2H^1H^2H^3 + H_0(H^0)^2 - H^AH_AH^0\,. \end{aligned} \tag{2.14}$$

If we replace the harmonic functions H in $\Sigma(H)$ by the charge vector Γ, the result is proportional to the Bekenstein-Hawking entropy $S(\Gamma)$ of a black hole with charge vector Γ: $\Sigma(\Gamma) = S(\Gamma)/\pi$.

The constants h in the harmonic functions are related to the asymptotic Kähler moduli as follows

$$h = -\frac{2}{\beta}\mathrm{Im}\frac{\bar{Z}_{\rm hol}\Omega}{|Z_{\rm hol}|}\,\Big|_\infty\,, \tag{2.15}$$

where $Z_{\rm hol}$ is the holomorphic central charge

$$Z_{\rm hol} = \langle \Gamma_{\rm tot}, \Omega_{\rm hol}\rangle\,. \tag{2.16}$$

Of the 8 components of h, only 6 are independent, corresponding to the asymptotic values of the 6 moduli $a_A,\ b_A$. Indeed, from the expressions above it follows that the h satisfy two constraints

$$\begin{aligned} \Sigma(h) &= \frac{1}{\beta^2}\,, \\ \langle h, Q_{\rm tot}\rangle &= 0\,. \end{aligned} \tag{2.17}$$

The metric of the multi-centered solution is given by

$$ds_4^2 = -\frac{1}{\beta^2\Sigma(H)}(dt+\omega)^2 + \beta^2\Sigma(H)d\vec{x}^2\,, \tag{2.18}$$

where ω is a 1-form on $\mathbf{R^3}$ that satisfies

$$\star_3 d\omega = \beta^2\langle dH, H\rangle = \beta^2\left(-H_0 dH^0 + H_A dH^A - H^A dH_A + H^0 dH_0\right)\,, \tag{2.19}$$

where the Hodge star $\star_3$ is to be taken with respect to the flat metric on $\mathbf{R^3}$. The integrability condition for the existence of ω leads to constraints on the positions of the centers:

$$\sum_t \frac{\langle Q_s, Q_t\rangle}{|x_s - x_t|} + \langle Q_s, h\rangle = 0\,. \tag{2.20}$$

An important condition for the existence of the supergravity solution is that, when the above conditions are imposed, the function $\Sigma(H)$ should be real everywhere. Multicenter solutions whose charges are non-parallel also carry angular momentum given by

$$\vec{J} = \frac{1}{2}\sum_{s<t}\langle\Gamma_s,\Gamma_t\rangle\frac{\vec{x}_s - \vec{x}_t}{|\vec{x}_s - \vec{x}_t|}\,. \tag{2.21}$$

In the special case of only 2 centers, the constraint on the distance a between the centers simplifies to

$$a = \frac{\langle Q_1, Q_2\rangle}{\langle Q_2, h\rangle}\,, \tag{2.22}$$

while the angular momentum is

$$J_z = \frac{1}{2}\langle\Gamma_1,\Gamma_2\rangle\,, \tag{2.23}$$

where we have chosen the z-axis to run in the direction from the second to the first center.

The solution for the scalar moduli reads

$$z^A = \frac{\frac{\partial\Sigma(H)}{\partial H_A} - iH^A}{\frac{\partial\Sigma(H)}{\partial H_0} + iH^0}\,. \tag{2.24}$$

More explicitly, splitting z^a into real and imaginary parts $z^A = a_A + ib_A,\ A = 1,2,3$ one finds

$$\begin{aligned} a_A &= -\frac{H^A}{H^0} + \frac{L}{2x_A H^0} \\ b_A &= \frac{\Sigma}{2x_A}\,. \end{aligned} \tag{2.25}$$

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The gauge fields are given by

$$\begin{aligned}\mathcal{A}^0 &= \frac{1}{\beta}\frac{\partial \ln \Sigma(H)}{\partial H_0}(dt+\omega) + \mathcal{A}^0_D\,,\\ \mathcal{A}^A &= -\frac{1}{\beta}\frac{\partial \ln \Sigma(H)}{\partial H_A}(dt+\omega) + \mathcal{A}^A_D\,,\end{aligned} \tag{2.26}$$

where the Dirac parts $\mathcal{A}^I_D$ have to satisfy

$$\star_3 d\mathcal{A}^I_D = dH^I\,. \tag{2.27}$$

More explicitly, one finds

$$\begin{aligned}\mathcal{A}^0 &= -\frac{1}{\beta}\frac{L}{\Sigma^2}(dt+\omega) + \mathcal{A}^0_D\,,\\ \mathcal{A}^A &= \frac{1}{\beta}\frac{6D_{ABC}x_Bx_C - H^AL}{H^0\Sigma^2}(dt+\omega) + \mathcal{A}^A_D\,.\end{aligned} \tag{2.28}$$

These quantities can be worked out a little more explicitly as

$$\Sigma = \sqrt{-4H_0H^1H^2H^3 - 4H^0H_1H_2H_3 + (H^IH_I)^2 - 2\sum_I (H_I)^2(H^I)^2}\,, \tag{2.29}$$

$$\begin{aligned}a_A &= \frac{H_0H^0 + H_AH^A - \sum_{B\neq A} H_BH^B}{6D_{ABC}H^BH^C - 2H_AH^0}\,,\\ b_A &= \frac{\Sigma}{6D_{ABC}H^BH^C - 2H_AH^0}\,,\end{aligned} \tag{2.30}$$

$$\begin{aligned}\mathcal{A}^0 &= \frac{1}{\beta\Sigma^2}\left(H^0\left(H_IH^I - 2H_0H^0\right) - 2D_{ABC}H^AH^BH^C\right)(dt+\omega) + \mathcal{A}^0_D\,,\\ \mathcal{A}^A &= -\frac{1}{\beta\Sigma^2}\left(H^A\left(H_IH^I - 2H^AH_A\right) - 6D_{ABC}H_BH_CH^0\right)(dt+\omega) + \mathcal{A}^A_D\,.\end{aligned} \tag{2.31}$$

We will also consider the effect of large gauge transformations of the B-field, under which the B-field shifts with a harmonic form. Gauge invariance requires that this is accompanied by a shift in the worldvolume flux, resulting in a transformation of the charge vector:

$$B \to B + S \qquad \Gamma \to e^S\Gamma\,. \tag{2.32}$$

In the 4D effective theory, the above transformation is induced by a symplectic transformation

$$X^A \to X^A + S^AX^0. \tag{2.33}$$

Taking charge quantization into account, S should be restricted to be an element of the integer cohomology. Large gauge transformations change the boundary conditions at infinity and, in the dual conformal field theory, have the effect of inducing a spectral flow [47, 5, 48].

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2.3 Solutions for polar states

We will now describe a particular set of 2-centered solutions where the centers are stacks of D6 and anti-D6 branes with worldvolume fluxes turned on. We will also show that for these configurations no single-centered solutions with the same total charge exist. In the language of [1], they correspond to polar states and are described by attractor flow trees as we will review in paragraph 2.4.

We will consider here two classes of polar states: the first class carries no D0-brane charge and has four net D4-D2 charges p^1, p^2, p^3, q_1. These are the configurations (1.4) with $m = 0$. By performing a spectral flow transformation of the form (2.32) we will obtain a second class of solutions ($m \neq 0$ in (1.4)) which carry the above four charges as well as D0-brane charge q_0. In section 3 we will show that these two classes of configurations, after a U-duality transformation, give rise to smooth 'fuzzball' solutions placed in a Taub-NUT background. The solutions without D0-charge will map to fuzzball solutions with D1-charge and D5-charge in Taub-NUT space while the solutions carrying D0-charge will map to fuzzball solutions with D1-D5 charge and momentum P in Taub-NUT.

2.3.1 Configurations without D0-charge

The first class of solutions we want to consider consists of a stack of n D6 branes and a stack of n anti-D6 branes. Each stack of branes has $\mathrm{U}(n) = \mathrm{U}(1) \times \mathrm{SU}(n)$ gauge fields living on the worldvolume. We will turn on worldvolume fluxes lying in the $\mathrm{U}(1)$ part so that each stack carries lower-dimensional D-brane charges as well. The fluxes we will turn on are characterized by three numbers which, for later convenience, we will label N_K, N_1, N_5. The charges at the centers are

$$\begin{aligned} \Gamma_1 &= -n\, e^{-\frac{N_K}{n}\omega_1} = (-n, N_K, 0, 0, 0, 0, 0, 0)\ , \\ \Gamma_2 &= n\, e^{\frac{N_1}{n}\omega_2 + \frac{N_5}{n}\omega_3} = \left(n, 0, N_1, N_5, \frac{N_1 N_5}{n}, 0, 0, 0\right)\ . \end{aligned} \tag{2.34}$$

In the quantum theory, charge quantization restricts n, N_K, N_1, N_5 to be integers and n to be a divisor of $N_1 N_5$. These configurations carry 4 nonzero net charges p^1, p^2, p^3, q_1:

$$\Gamma_{\rm tot} = \left(0, N_K, N_1, N_5, \frac{N_1 N_5}{n}, 0, 0, 0\right)\ . \tag{2.35}$$

We will choose coordinates on $\mathbf{R}^3$ such that the first center Γ_1 is located at the origin and Γ_2 lies on the positive z-axis at $z = a$. The harmonic functions are

$$\begin{aligned} H^0 &= h^0 - \frac{Q_n}{r} + \frac{Q_n}{r_+}\,, & H_0 &= h_0\,, \\ H^1 &= h^1 + \frac{Q_K}{r}\,, & H_1 &= h_1 + \frac{Q_1 Q_5}{Q_n r_+}\,, \\ H^2 &= h^2 + \frac{Q_1}{r_+}\,, & H_2 &= h_2\,, \\ H^3 &= h^3 + \frac{Q_5}{r_+}\,, & H_3 &= h_3\,. \end{aligned} \tag{2.36}$$

We have defined r_+ to be the radial distance to the second center:

$$r_+ \equiv \sqrt{r^2 + a^2 - 2ar\cos\theta}\,. \tag{2.37}$$

From now on, we will choose the normalization constant β in (2.1) to be

$$\beta = \frac{1}{\sqrt{2}}. \tag{2.38}$$

Using (2.8), the normalizations in the harmonic functions are then given by

$$\begin{aligned} Q_n &= \frac{1}{2}\sqrt{\alpha'}gn & Q_K &= \frac{(2\pi)^2(\alpha')^{\frac{3}{2}}g}{2V_{T_1}}N_K \\ Q_1 &= \frac{(2\pi)^2(\alpha')^{\frac{3}{2}}g}{2V_{T_2}}N_1 & Q_5 &= \frac{(2\pi)^2(\alpha')^{\frac{3}{2}}g}{2V_{T_3}}N_5 \end{aligned} \tag{2.39}$$

where g is the 10D string coupling constant.

We can simplify the form of the solution by picking convenient values for the asymptotic moduli and correspondingly the constants h. We will choose six of the constants to be

$$h_0 = -1; \qquad h^1 = h^2 = h^3 = 1; \qquad h_2 = h_3 = 0\,. \tag{2.40}$$

The remaining constants h^0, h_1 are then fixed by the constraints (2.17) to be

$$h_1 = -h^0 = \frac{Q_1Q_5}{Q_nQ_K}\,. \tag{2.41}$$

From (2.30) we see that this choice of harmonic constants corresponds to turning on asymptotic B-field on T_1 but not on $T_2,\ T_3$.

The constraint (2.22) on the distance between the centers reads

$$a = \frac{Q_KQ_1Q_5}{Q_n^2 - Q_1Q_5}\,. \tag{2.42}$$

The solution carries angular momentum given by (2.23):

$$J_z = \frac{N_KN_1N_5}{2n}. \tag{2.43}$$

One can then find the explicit expressions for the metric, scalar fields and U(1) fields from (2.18), (2.30), (2.31). For configurations where $H_2 = H_3 = 0$, the expression (2.29) for Σ simplifies to

$$\Sigma = \sqrt{-4H_0H^1H^2H^3 - (H_0H^0 - H_1H^1)^2}\,. \tag{2.44}$$

For the solution to the equations (2.19) and (2.27) for ω and the Dirac parts $\mathcal{A}_D^I$ one finds, using (2.42) and choosing convenient integration constants,

$$\begin{aligned} \omega &= \frac{Q_KQ_1Q_5}{2aQ_n}\left(\frac{r+a}{r_+} - 1\right)(\cos\theta - 1)d\phi\,, \\ \mathcal{A}_D^0 &= Q_n\left(-\cos\theta + \frac{r\cos\theta - a}{r_+}\right)d\phi\,, \end{aligned}$$

$$\begin{aligned}
\mathcal{A}^1_D &= Q_K \cos\theta d\phi\,,\\
\mathcal{A}^2_D &= Q_1 \frac{r\cos\theta - a}{r_+} d\phi\,,\\
\mathcal{A}^3_D &= Q_5 \frac{r\cos\theta - a}{r_+} d\phi\,.
\end{aligned} \tag{2.45}$$

2.3.2 Spectral flow and adding D0-charge

The second class of solutions we will be interested in is obtained from the ones considered above by a spectral flow transformation of the form (2.32) $\Gamma \to e^S\Gamma$. We can choose S such that the new configuration carries nonzero p^1, p^2, p^3, q_1 charges as well as D0-charge q_0, while keeping q_2 and q_3 zero. There is a one-parameter family of spectral flows S which does the job and which we will label by a parameter m:

$$S = -mN_K\omega_1 + mN_1\omega_2 + mN_5\omega_3\,. \tag{2.46}$$

When taking charge quantization into account, the parameter m could be fractional but such that m is a common multiple of $1/N_1, 1/N_5$ and $1/N_K$. The charges carried by the two centers are then the ones anticipated in (1.4) in the introduction:

$$\begin{aligned}
\Gamma_1 &= -ne^{-\left(m+\frac{1}{n}\right)N_K\omega_1 + mN_1\omega_2 + mN_5\omega_3}\,,\\
\Gamma_2 &= ne^{-mN_K\omega_1 + \left(m+\frac{1}{n}\right)N_1\omega_2 + \left(m+\frac{1}{n}\right)N_5\omega_3}\,,
\end{aligned} \tag{2.47}$$

and the total charge of the solution is

$$\Gamma_{\rm tot} = \left(0, N_K, N_1, N_5, \left(2m+\frac{1}{n}\right)N_1N_5, 0, 0, -m\left(m+\frac{1}{n}\right)N_KN_1N_5\right)\,. \tag{2.48}$$

The angular momentum of these configurations is independent of the parameter m and still given by (2.43). For $m = 0$ we recover the configurations discussed in the previous section.

The harmonic functions for this configuration are

$$\begin{aligned}
H^0 &= h^0 - \frac{Q_n}{r} + \frac{Q_n}{r_+}\,,\\
H_0 &= h_0 + \frac{(mn+1)(mn)^2Q_KQ_1Q_5}{Q_n^2 r} - \frac{(mn+1)^2mnQ_KQ_1Q_5}{Q_n^2 r_+}\,,\\
H^1 &= h^1 + \frac{(mn+1)Q_K}{r} - \frac{mnQ_K}{r_+}\,,\\
H_1 &= h_1 - \frac{(mn)^2Q_1Q_5}{Q_n r} + \frac{(mn+1)^2Q_1Q_5}{Q_n r_+}\,,\\
H^2 &= h^2 - \frac{mnQ_1}{r} + \frac{(mn+1)Q_1}{r_+}\,,\\
H_2 &= h_2 + \frac{(mn+1)mnQ_KQ_5}{Q_n r} - \frac{(mn+1)mnQ_KQ_5}{Q_n r_+}\,,\\
H^3 &= h^3 - \frac{mnQ_5}{r} + \frac{(mn+1)Q_5}{r_+}\,,\\
H_3 &= h_3 + \frac{(mn+1)mnQ_KQ_1}{Q_n r} - \frac{(mn+1)mnQ_KQ_1}{Q_n r_+}\,.
\end{aligned} \tag{2.49}$$

As before, we choose the asymptotic moduli such that $h_0 = -1$, $h^1 = h^2 = h^3 = 1$, $h_2 = h_3 = 0$. The remaining constants are determined by (2.17) to be

$$h_1 = -h^0 = \frac{(2mn+1)Q_1Q_5Q_n}{(mn+1)mnQ_KQ_1Q_5 + Q_KQ_n^2}\,. \tag{2.50}$$

For the constraint (2.22) on the distance one finds a rather complicated expression

$$\begin{aligned}\frac{1}{a} = \frac{1}{Q_KQ_1Q_5\left((mn+1)^2(mn)^2Q_1Q_5 + Q_n^2\right)} \times \\ \times\Big(Q_n^4 - Q_n^2\left(Q_1Q_5 + (mn+1)(2Q_1Q_5 - Q_K(Q_1+Q_5))\right) \\ +(mn+1)^2(mn)^2Q_1Q_5(Q_KQ_1 + Q_KQ_5 + Q_1Q_5)\Big)\,.\end{aligned} \tag{2.51}$$

2.4 Polarity and flow trees

We will now describe how our configurations fit within the zoo of four-dimensional multicenter BPS solutions, using the tools that were developed in [1]. Some well-founded conjectures put forth there will allow us to draw conclusions which are valid beyond the leading supergravity approximation. We will now review some relevant points from [1] to which we refer the reader for more details.

The configurations we are considering here correspond to four-dimensional 'polar' states. Mathematically, polar states can be seen as the constituents of the polar part of the black hole partition function as a generalized modular form. The full partition function can be reconstructed from the knowledge of the degeneracies of the polar states, which was at the core of deriving an OSV-type relation for D4-D2-D0 black holes in [1].

Physically, the fact that a configuration is polar means that no single-centered solutions with these charges exist. For polar configurations, one can show that the attractor flow equations that describe the radial evolution of the moduli fields always 'crash' at a regular[3] point in moduli space beyond which they cannot be continued. This means that a single-centered black hole solution cannot exist in the supergravity approximation. Furthermore, by appropriately choosing the asymptotic moduli, one can show that, at the point where the attractor flow crashes, all curvatures remain small, and hence this conclusion should not be modified by higher-derivative corrections to supergravity [49].

As discussed in [1] the relevant quantity for establishing whether a total charge system $\Gamma_{\rm tot}$ is polar is the 'reduced' D0 brane charge

$$\hat{q}_0 = q_0 - \frac{1}{2}D^{AB}q_Aq_B\,, \tag{2.52}$$

where $D^{AB} = (6D_{ABC}p^C)^{-1}$. If $\hat{q}_0 > 0$, the states are polar and no single centered black hole solutions carrying these charges exist. For our configurations without D0-charge (2.35) one obtains

$$D^{AB} = \frac{1}{2N_KN_1N_5}\begin{pmatrix} -N_K^2 & N_KN_1 & N_KN_5 \\ N_KN_1 & -N_1^2 & N_1N_5 \\ N_KN_5 & N_1N_5 & -N_5^2 \end{pmatrix}, \tag{2.53}$$

[3]Regular meaning that the Kähler form on the internal space lies within the Kähler cone.

and therefore

$$\hat{q}_0 = \frac{N_K N_1 N_5}{4n^2}. \tag{2.54}$$

This means that these states are polar if we choose positive fluxes on our branes. For $n = 1$, when there is only one D6 and one anti-D6 brane, $\hat{q}_0$ reaches its maximal value for given p^1, p^2, p^3 charge. The quantity $\hat{q}_0$ is invariant under spectral flow transformations (2.32), hence our charge configurations with D0 charge (2.48) are also polar with $\hat{q}_0$ still given by (2.54).

Even if no single-centered solution exists, there can still be a BPS state carrying the desired charges which is realized as a multicentered configuration[4]. A proposed criterion to verify whether such a BPS state exists is whether there is an 'attractor flow tree' for the given charge. This proposal is called the 'split attractor flow conjecture' and has been argued to establish the existence of the BPS state beyond the supergravity approximation. An attractor flow tree is a graph in moduli space which starts at the background value of the moduli and follows the single center attractor flow until it hits a wall of marginal stability where it becomes energetically possible for the total charge to split into two constituents. There the flow splits in two parts corresponding to the single centered flows of the constituents. This process is repeated until one ends up at the attractor points for all the centers of the configuration.

We therefore now inspect the existence of flow trees for our charge configurations in order to be able to infer the existence of the corresponding BPS state. We will show that the single centered flow reaches a wall of marginal stability at a point z_{split} in moduli space before reaching the crash point z_0, where the single centered flow ends. At the marginal stability wall, the flow branches into two flows representing the D6 and anti D6 centers which reach their attractor points without encountering any more marginal stability walls. A schematic depiction of the split flow is given in figure 1.

A crucial simplification is that, doing the spectral flow transformation (2.32), we can equivalently examine the flow tree for a charge $e^S\Gamma$ at a shifted B-value $B + S$. When, by shifting the asymptotic value of the B-field, one does not cross any walls of marginal stability, we can simply fix the asymptotic B-field to a convenient value and choose a charge vector $e^S\Gamma$ such that the analysis becomes simple. This will be possible for our configurations, provided that we choose the background Kähler moduli large enough. The reason for this is that walls of marginal stability between two charges can only run all the way to infinity for a 'core-halo pair' of D-branes (Halos can only carry D2-D0 brane charge, any other charge configuration will automatically be a core, see [1] for definitions and a more in-depth treatment of these concepts). Here, we are luckily always dealing with core constituents. From now on, we will take the asymptotic B-field to be zero and choose an appropriate charge vector $e^S\Gamma$.

We can pick a charge representative by giving some convenient value to the spectral flow parameter m in our general charge configuration (2.48). We will take it to have the

[4]Note that constituents need not be 'regular' black hole solutions, but can also be realized as 'empty' holes where the center has zero entropy.

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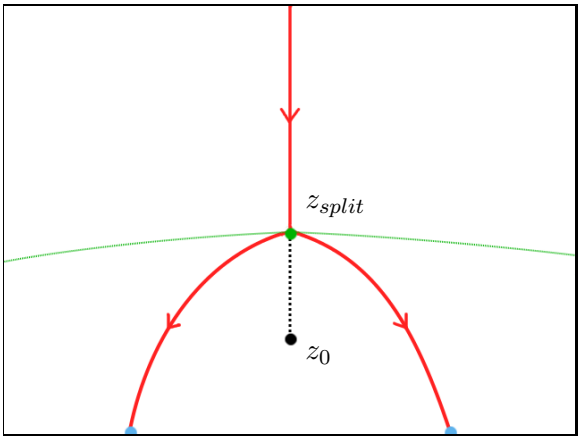

Figure 1: Schematic drawing of the split flow tree for our representative charge system. The flow coming in from the top (red line) reaches the wall of marginal stability (green line) at the splitpoint z_{split} (green) before it would reach the crash point z_0 (black). One also sees the single flows for each center starting from the split point until they reach the boundary of moduli space (blue).

value[5] $m = -\frac{1}{2n}$. This leads us to the total charge

$$\Gamma_{\rm tot} = \left(0, N_K, N_1, N_5, 0, 0, 0, \frac{N_K N_1 N_5}{4n^2}\right) . \tag{2.55}$$

This obviously is a pure D4-D0 system. As discussed above, we choose our background modulus to have purely imaginary and very large values, $z_\infty = (iy^1_\infty, iy^2_\infty, iy^3_\infty)$. The single centered flow runs along the imaginary z-axes until the crash point is reached where the holomorphic central charge (2.16) vanishes. This happens at the point

$$z_0 = i\sqrt{\frac{2\hat{q}_0}{6N_K N_1 N_5}}(N_K, N_1, N_5) = i\frac{1}{\sqrt{12n^2}}(N_K, N_1, N_5)\,. \tag{2.56}$$

Next one can check whether the flow hits a wall of marginal stability. This is per definition the locus where the phases of the central charges of the two centers align. The charges at the centers read

$$\begin{aligned}
\Gamma_1 &= \left(-n, \frac{N_K}{2}, \frac{N_1}{2}, \frac{N_5}{2}, -\frac{N_1 N_5}{4n}, -\frac{N_K N_5}{4n}, -\frac{N_K N_1}{4n}, \frac{N_K N_1 N_5}{8n^2}\right) , \\
\Gamma_2 &= \left(n, \frac{N_K}{2}, \frac{N_1}{2}, \frac{N_5}{2}, \frac{N_1 N_5}{4n}, \frac{N_K N_5}{4n}, \frac{N_K N_1}{4n}, \frac{N_K N_1 N_5}{8n^2}\right) .
\end{aligned} \tag{2.57}$$

One easily sees that the real parts of the central charges are equal, whereas the imaginary parts have opposite signs. Thus, the wall will be hit when $\mathrm{Im}(Z_1) = \mathrm{Im}(Z_2) = 0$. One finds

$$z_{\rm split} = i\sqrt{\frac{3}{4n^2}}(N_K, N_1, N_5)\,. \tag{2.58}$$

[5] As the flow tree analysis takes place within supergravity, we can ignore charge quantization restrictions for the moment.

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As $\sqrt{\frac{3}{4n^2}} > \sqrt{\frac{1}{12n^2}}$ this means that the wall of marginal stability is always reached before the single flow crashes. The single centered flows for the fluxed D6 brane centers terminate at the boundary of moduli space in the supergravity approximation. Nevertheless they correspond to states in the BPS spectrum of string theory and higher derivative corrections are expected to yield regular attractor points.

A further simple check also shows that the necessary stability criterion [50, 1] $\langle \Gamma_1, \Gamma_2 \rangle \cdot (\arg(Z_1) - \arg(Z_2)) > 0$ is met. This shows that one indeed reaches the wall from the side where the single brane is stable and crosses to the side where the brane decays into a bound state. The condition can be interpreted as ensuring that tachyonic strings would be present between the two constituent branes on the 'stable' side, in this case above the wall, such that a bound state is formed after tachyon condensation.

3. U-duality and fuzzballs in Taub-NUT

In this section, we would like to make contact between the polar solutions constructed above and various horizonless supertube solutions in five noncompact dimensions that are central to the fuzzball proposal advocated by Mathur and collaborators. As a first step, we will make a duality transformation to a type IIB frame such that the charges and dipole moments carried by our solutions are the same as the ones carried by the supertubes.

Let us briefly review these configurations. Fuzzball solutions in five noncompact dimensions can be seen as Kaluza-Klein (KK) monopole[6] supertubes where the KK monopole charge is sourced along a contractible curve in 4 noncompact directions. One of the compact directions, which will become S^4 in our conventions (recall that we had denoted $T_1 = S^4 \times S^5$), is a Taub-NUT circle which pinches off at every point of the curve. By adding flux to the KK-monopole, one can source the charge of D1 and D5-branes wrapped around the S^4 circle. For a circular curve, one can place this configuration in a Taub-NUT space with a different Taub-NUT circle, S^5 in our conventions, and interpolate between five and four dimensions by varying the size of S^5. We will show that the four-dimensional configurations obtained in this manner are U-dual to the D6-anti D6 polar solutions we discussed above.

3.1 U-duality to a type IIB frame

Let us first describe a U-duality transformation to a type IIB frame such that STU-model solutions lift to configurations carrying the charges described above. We will go to a duality frame where p^0 becomes a Kaluza-Klein monopole charge with Taub-NUT circle S^4, p^1 becomes a Kaluza-Klein monopole charge with Taub-NUT circle S^5, p^2 becomes the charge of a $D1$-brane wrapped on S^4 and p^3 becomes the charge of a $D5$-brane wrapped on $S^4 \times T_2 \times T_3$. This is accomplished by making a U-duality transformation consisting of a T-duality along S^4, followed by S-duality and 4 T-dualities along $T_1 \times T_3$, as illustrated in table 2.

[6]Recall that a Kaluza-Klein monopole in 10D is a 5+1-dimensional object whose transverse 4-dimensional space has Taub-NUT geometry or, in the case of several centers, a Gibbons-Hawking space.

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IIA (frame A)		IIB		IIB		IIB (frame B)
D6 (T^6)		D5		NS5		KK5 ($S^5 \times T_2 \times T_3$)
D4 ($T_2 \times T_3$)	T (S^4)	D5	S	NS5	T (S^4, S^5, T_3)	KK5 ($S^4 \times T_2 \times T_3$)
D4 ($T_1 \times T_3$)	$\longrightarrow$	D3	$\longrightarrow$	D3	$\longrightarrow$	D1 (S^4)
D4 ($T_1 \times T_2$)		D3		D3		D5 ($S^4 \times T_2 \times T_3$)

Table 2: U-duality transformation from frame A to frame B

q_0	P (S^4)	p^0	KK5 ($S^5 \times T_2 \times T_3$)
q_1	P (S^5)	p^1	KK5 ($S^4 \times T_2 \times T_3$)
q_2	D5 ($S^5 \times T_2 \times T_3$)	p^2	D1 (S^4)
q_3	D1 (S^5)	p^3	D5 ($S^4 \times T_2 \times T_3$)

Table 3: 10D origin of the charges in frame B

This new duality frame will be denoted 'frame B'. In this frame, the vector multiplet scalars z^1, z^2, z^3 represent the complex structure modulus of T_1, the 4D axion-dilaton and the (complexified) Kähler modulus of T_1 respectively. The U(1) fields $\mathcal{A}^0$ and $\mathcal{A}^1$ are Kaluza-Klein gauge fields from the metric components $g_{\mu 4}$ and $g_{\mu 5}$ respectively, while $\mathcal{A}^2$ and $\mathcal{A}^3$ arise from the RR two form components $C_{\mu 4}$ and $C_{\mu 5}$. The 10-dimensional origin of the full set of charges in this frame is given in table 3.

In frame B, our first class of polar solutions with charges (2.34) corresponds to two stacks of n KK monopoles and anti-KK monopoles with Taub-NUT circle S^4 carrying flux-induced charges of D1, D5, momentum and KK monopoles wrapped on the S^4 circle. The more general solutions (2.47) obtained by spectral flow carry momentum along S^4 as well. Such solutions will be smooth, and, as we will show, have the interpretation of KK monopole supertubes embedded in Taub-NUT space.

3.2 Lifting general multicenter solutions

In order to see what our solutions look like in frame B from the 10-dimensional point of view, we need to know the reduction formulas of type IIB on a six-torus to the four-dimensional STU-model action (2.1) such that the 4D charges have the interpretation given in table 3. This is worked out in detail in appendix A.

The metric of a general 4D multicentered solution lifts to a 10D geometry where the T_1 torus is nontrivially fibered over the 4D base:

$$\begin{aligned} ds_{10}^2 &= \frac{1}{\sqrt{b^2 b^3}} ds_4^2 + \sqrt{b^2 b^3}\, \mathcal{M}_{mn} (dx^m + \mathcal{A}^{m-4})(dx^n + \mathcal{A}^{n-4}) + \sqrt{\frac{b^2}{b^3}} ds^2_{T_2 \times T_3}\,, \\ ds_4^2 &= -\frac{2}{\Sigma}(dt + \omega)^2 + \frac{\Sigma}{2} d\vec{x}^2\,, \\ \mathcal{M}_{mn} &= \frac{1}{b^1} \begin{pmatrix} (a^1)^2 + (b^1)^2 & -a^1 \\ -a^1 & 1 \end{pmatrix}, \qquad m, n = 4, 5\,. \end{aligned} \tag{3.1}$$

The dilaton and RR two-form are given by

$$\begin{aligned}
e^{2\Phi^{(10)}} &= \frac{b^2}{b^3}\,,\\
C^{(10)} &= \frac{1}{2}C_{\mu\nu}dx^\mu dx^\nu + a^3(dx^4-\mathcal{A}^0)\wedge(dx^5-\mathcal{A}^1)\\
&\quad -dx^4\wedge\mathcal{B}^2 - dx^5\wedge\mathcal{A}^3 + \frac{1}{2}(\mathcal{A}^0\wedge\mathcal{B}^2+\mathcal{A}^1\wedge\mathcal{A}^3)\,,\\
da^2 &= -(b^2)^2\star F\,,\\
F &= dC + \frac{1}{4}(\mathcal{A}^0\wedge\mathcal{G}^2 + \mathcal{B}^2\wedge\mathcal{F}^0 + \mathcal{A}^1\wedge\mathcal{F}^3 + \mathcal{A}^3\wedge\mathcal{F}^1)\,.
\end{aligned} \tag{3.2}$$

where the Hodge $\star$ is to be taken with respect to the 4D metric ds_4^2.

It will be useful to rewrite the metric in the form of a lifted solution of 6D supergravity as in [51, 9, 48], where the 6D part of the metric is written as a fibration over a 4D Gibbons-Hawking base space. If both p^0 and p^1 are nonzero, both the S^4 and S^5 are nontrivially fibered, and we can choose either circle to be the fibre in the Gibbons-Hawking geometry. Here, we will choose the S^5 to be this fibre, so that the Gibbons-Hawking base space is spanned by the coordinates (r,θ,ϕ,x^5). The metric can be rewritten in the form

$$\begin{aligned}
ds^2 &= -\frac{1}{HF}(dt+k)^2 + \frac{F}{H}\left(dx^4 - s - \frac{1}{F}(dt+k)\right)^2 + H ds^2_{\rm GH} + \sqrt{\frac{x_2}{x_3}}ds^2_{T_2\times T_3}\,,\\
ds^2_{\rm GH} &= \frac{1}{H^1}(dx^5+\mathcal{A}^1_D)^2 + H^1 d\mathbf{x}^2\,.
\end{aligned} \tag{3.3}$$

where we have defined

$$\begin{aligned}
F &= \left(\frac{H_2H_3}{H^1} - H_0\right)\,,\\
H &= \frac{\sqrt{x_2x_3}}{H_1}\,,\\
k &= \omega + \frac{LH^1 - 2x_2x_3}{2H^0(H^1)^2}(dx^5+\mathcal{A}^1_D)\\
&= \omega + \frac{1}{2H_1}\left(H_IH^I - 2H_1H^1 - \frac{2H_0H_2H_3}{H^1}\right)(dx^5+\mathcal{A}^1_D)\,,\\
s &= -\mathcal{A}^0_D + \frac{H^0}{H^1}(dx^5+\mathcal{A}^1_D)\,.
\end{aligned} \tag{3.4}$$

We will now use these expressions to find the lift of our four-dimensional polar configurations.

3.3 Lift of polar states without D0 charge

We will first discuss the lift of our configurations (2.34) that do not contain D0 charge in frame A. In frame B these correspond, according to table 3, to two stacks of n KK monopoles and anti-KK monopoles with Taub-NUT circle S^4 which carry flux-induced D1, D5 and KK monopole charges wrapped on the S^4 circle. We will now show that, from a 10D point of view, these charges precisely correspond to the Kaluza-Klein monopole supertubes in Taub-NUT space that were constructed by Bena and Kraus in [20].

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The harmonic functions of the solution are given by (2.36), (2.40), (2.41), where the normalizations in the current duality frame should be taken to be, according to (2.8),

$$Q_n = \frac{nR_4}{2}\,, \qquad Q_K = \frac{N_K R_5}{2}\,, \qquad (3.5)$$
$$Q_1 = \frac{(2\pi)^4 g\alpha'^3}{2R_5 V_{T_2\times T_3}} N,\,. \qquad Q_5 = \frac{g\alpha'}{2R_5} N_5\,.$$

The constraint on the distance between the centers (2.42) can also be written as

$$Q_n = \sqrt{Q_1 Q_5 \tilde{H}^1}\,, \qquad (3.6)$$

with $\tilde{H}^1 = 1 + \frac{Q_K}{a}$.

We find the lift of this class of solutions to 10 dimensions in duality frame B by plugging these expressions into (3.3). Making a coordinate transformation $x^4 \to x^4 + t$, the metric becomes

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H^2 H^3}}\left[-(dt+k)^2 + (dx^4 - s - k)^2\right] + \sqrt{H^2 H^3} ds^2_{\rm TN} + \sqrt{\frac{H^2}{H^3}} ds^2_{T_2\times T_3}\,, \\ ds^2_{\rm TN} &= \frac{1}{H^1}(R_5 d\psi + Q_K \cos\theta d\phi)^2 + H^1 d\mathbf{x}^2\,. \end{aligned} \qquad (3.7)$$

where we have defined the angle ψ as $x^5 = R_5\psi$. From the ten-dimensional point of view, the constraint (3.6) on the distance between the centers arises from requiring smoothness of the metric [20], while the condition that Σ is real implies the absence of closed timelike curves [39].

The one-forms k and s have components along ϕ and ψ and, using the distance constraint (3.6), can be written as

$$\begin{aligned} k_\psi &= \frac{R_5 Q_n Q_K}{2ar r_+ \tilde{H}^1 H^1}\left[r_+ - r - a - \frac{2ar}{Q_K}\right]\,, \quad k_\phi = \frac{Q_n Q_K}{2ar_+\tilde{H}^1}\left[r_+ - r - a + \frac{r-a-r_+}{H^1}\cos\theta\right]\,, \\ s_\psi &= \frac{R_5 Q_n}{r r_+ H^1}\left[r - r_+ - \frac{r r_+}{Q_K \tilde{H}^1}\right]\,, \qquad s_\phi = \frac{Q_n}{r_+}\left[a + \frac{r_+ - r - \frac{r_+}{\tilde{H}^1}}{H^1}\cos\theta\right]\,. \end{aligned} \qquad (3.8)$$

Using (3.2) one can show that the dilaton and RR three-form take the form

$$\begin{aligned} e^{2\Phi} &= \frac{H^2}{H^3}\,, \\ F^{(3)} &= d\left[\frac{1}{H^2}(dt+k)\wedge(dx^4 - s - k)\right] - \star_4 d(H^3)\,, \end{aligned} \qquad (3.9)$$

where the Hodge star $\star_4$ is to be taken with respect to the Taub-NUT metric ds^2_{TN}.

As we have argued, the above solutions represent the lift of a two-centered KK-monopole anti-monopole system in frame B (or a D6 anti-D6 system in frame A), where the Taub-NUT circle for these KK monopoles is the S^4. The KK monopoles sit at a radial distance r_+ while the anti-monopoles sit at the origin. At the position of these centers, the S^4 circle should pinch off. This is not so obvious in the 10D form of the metric (3.7), so let

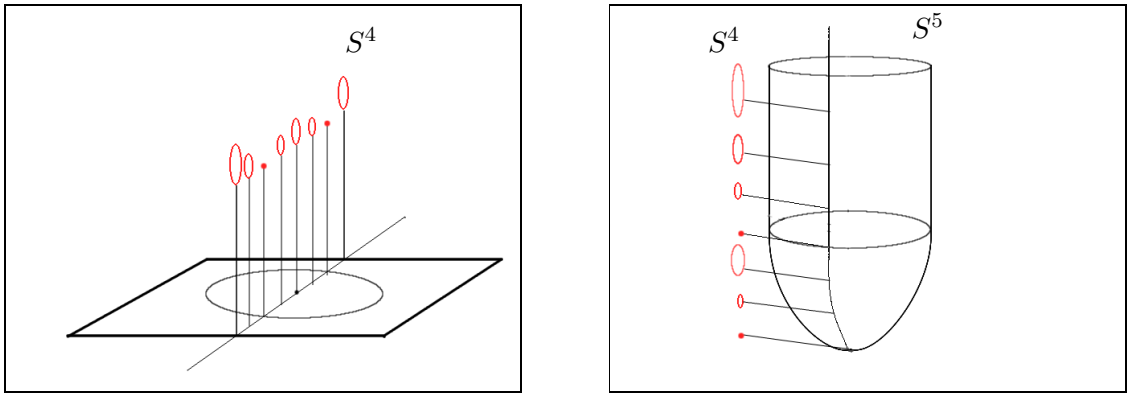

Figure 2: Left: The black circle represents a KK monopole supertube with a circular profile of radius a in 5 dimensions. At every point of the curve, the internal circle S^4 (drawn in red) pinches off to zero size. Right: After placing another KK monopole wrapped on S^4 in the origin, the asymptotic geometry becomes $\mathbf{R}^4 \times S^5$. As argued in the text, the S^4 circle pinches off along the curve as well as in the origin.

us illustrate this point in more detail here. The coefficient in front of the $(dx^4)^2$ term in the metric (3.7) is $1/\sqrt{H^2H^3}$. This factor goes to zero at $r = r_+$ but stays finite at $r = 0$, so it is not obvious that there is a KK anti-monopole source at the origin. Nevertheless, there should be such a source since the total KK monopole charge has to balance out, and it should be located at the origin because of symmetry reasons. The resolution to this puzzle lies in the fact that the six-dimensional metric still contains a factor of the six-dimensional dilaton $\mathrm{e}^{\Phi^{(6)}}$. This factor is given by $\mathrm{e}^{\Phi^{(6)}} = \frac{1}{b_2 b_3}$, and hence the factor that measures the size of the S^4 is $b_2 b_3/\sqrt{H^2H^3}$. One can easily check that this factor indeed goes to zero both in $r = 0$ and $r = r_+$. This is illustrated in figure 2.

These are precisely the solutions constructed by Bena and Kraus [20][7]. They represent Kaluza-Klein monopole supertubes which have been embedded into a Taub-NUT space which has the asymptotic spatial geometry $\mathbf{R}^3 \times S^5$. By varying the radius R_5 of the circle S^5 we can interpolate between solutions in 4 and in 5 noncompact dimensions; this procedure goes under the name of the '4D-5D connection' [28, 29]. The 5D solutions one obtains in this way are highly symmetric fuzzball solutions where the curve that defines the supertube is circular.

3.4 4D-5D connection and 5D fuzzball geometries

Let us illustrate this in more detail. We take the $R_5 \to \infty$ limit keeping the following quantities fixed:

$$2rR_5 \equiv \tilde{r}^2\,, \qquad 2aR_5 \equiv \tilde{a}^2/n^2\,. \tag{3.10}$$

[7]To make contact with the conventions in [20], one has to make a further coordinate transformation $\phi \to -\phi, \theta \to \pi - \theta$.

After taking this limit, the p^1 charge N_K of our configuration becomes a deficit angle and one obtains a configuration embedded in an orbifold space $\mathbf{R}^4/\mathbf{Z}_{N_K}$. We will therefore specialize to the case $N_K = 1$ from now on, so that we obtain solutions in asymptotically flat space. We define charges $\tilde{Q}_1, \tilde{Q}_5$ which remain finite in the limit (3.10) and are the correctly normalized D1 and D5-brane charges in 5 noncompact dimensions:

$$\begin{aligned} \tilde{Q}_1 &= 2R_5 Q_1 = \frac{g(2\pi)^4 \alpha'^3 N_1}{V_{T_2\times T_3}}\,, \\ \tilde{Q}_5 &= 2R_5 Q_1 = g\alpha' N_5\,. \end{aligned} \tag{3.11}$$

The constraint (2.42) on the distance between the centers then reduces to

$$R_4 = \frac{\sqrt{\tilde{Q}_1 \tilde{Q}_5}}{\tilde{a}}\,. \tag{3.12}$$

The solution (3.8), (3.9) can, in this limit, be written as a fuzzball solution with a circular profile function [12 – 15]:

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H^2 H^3}}\left[-(dt+k)^2 + (dx^4 - s - k)^2\right] + \sqrt{H^2 H^3}\, d\mathbf{x}^2 + \sqrt{\frac{H^2}{H^3}}\, ds^2_{T_2\times T_3}\,, \\ e^{2\Phi} &= \frac{H^2}{H^3}\,, \\ F^{(3)} &= d\left[\frac{1}{H^2}(dt+k)\wedge(dx^4 - s - k)\right] - \star_4 d(H^3)\,, \end{aligned} \tag{3.13}$$

where the harmonic functions are given by

$$\begin{aligned} H^2 &= 1 + \frac{\tilde{Q}_5}{L}\int_0^L \frac{dv}{|\mathbf{x}-\mathbf{F}|^2}\,, \\ H^3 &= 1 + \frac{\tilde{Q}_5}{L}\int_0^L \frac{|\dot{\mathbf{F}}|^2 dv}{|\mathbf{x}-\mathbf{F}|^2}\,, \end{aligned} \tag{3.14}$$

and the one-foms k, s take the form

$$\begin{aligned} s &= \frac{\tilde{Q}_5}{L}\int_0^L \frac{dv F^a}{|\mathbf{x}-\mathbf{F}|^2} dx^a\,, \\ d(s+k) &= -\star_4 ds\,. \end{aligned} \tag{3.15}$$

Here, $\mathbf{x}$ represents Cartesian coordinates on $\mathbf{R}^4$ which, in terms of the coordinates $\tilde{r}, \theta, \phi, \psi$ introduced earlier, are given by

$$\begin{aligned} x^1 &= \tilde{r}\cos\frac{\theta}{2}\cos\left(\psi + \frac{\phi}{2}\right)\,, & x^3 &= \tilde{r}\sin\frac{\theta}{2}\cos\left(\psi - \frac{\phi}{2}\right)\,, \\ x^2 &= \tilde{r}\cos\frac{\theta}{2}\sin\left(\psi + \frac{\phi}{2}\right)\,, & x^3 &= \tilde{r}\sin\frac{\theta}{2}\sin\left(\psi - \frac{\phi}{2}\right)\,. \end{aligned} \tag{3.16}$$

The profile function $\mathbf{F}(v)$ describes a circular profile in the $x^1 - x^2$ plane:

$$\begin{aligned} F^1 &= \frac{\tilde{a}}{n}\cos\frac{2\pi n}{L}v, & F^3 &= 0\,, \\ F^2 &= \frac{\tilde{a}}{n}\sin\frac{2\pi n}{L}v, & F^4 &= 0\,. \end{aligned}$$

where $L \equiv \frac{2\pi\tilde{Q}_5}{R_4}$. The averaged length of the tangent vector to the profile should be proportional to the D1-brane charge:

$$Q_1 = \frac{Q_5}{L}\int_0^L |\dot{\mathbf{F}}|^2 dv\,. \tag{3.17}$$

As a consistency check, one can easily see that this is the case using the constraint (3.12).

Let us also discuss how the 5D angular momenta are related to quantum numbers in 4D. Solutions in five noncompact dimensions can have 2 independent angular momenta J_{12} in the $x^1 - x^2$-plane and J_{34} in the $x^3 - x^4$-plane. These are related to the R-symmetry generators J^3 and $\bar{J}^3$ in the dual CFT as $J_{12} = -(J^3 + \bar{J}^3),\ J_{12} = -(J^3 - \bar{J}^3)$. From the parametrization (3.16) we see that J^3 comes from a linear momentum in four dimensions while $\bar{J}^3$ is proportional to the four-dimensional angular momentum J_z. This leads to the dictionary between the charges that was anticipated in 1.2. More specifically, the solutions above have $J_{12} = \frac{N_1 N_5}{n}, J_{34} = 0$, so that

$$J^3 = \bar{J}^3 = -\frac{N_1 N_5}{2n}. \tag{3.18}$$

3.5 Spectral flow and fuzzball solutions with momentum

In paragraph 2.3.2, we considered solutions that were obtained by a spectral flow transformation labeled by a parameter m that had the effect of adding D0-charge (2.48). In the dual frame B, these will carry nonzero momentum charge P on the S^4 circle. The harmonic functions and constraint on the distance were given in (2.49), (2.51). When we take the special case $Q_1 = Q_5$, substituting in (3.3) gives a solution with constant dilaton which can be embedded in minimal 6-dimensional supergravity [51]. This solution precisely matches the solutions constructed in [22] representing fuzzball geometries with momentum placed in a Taub-NUT space.

We can again take the 5D limit $R_5 \to \infty$ as discussed above. Taking again $N_K = 1$ to get solutions in flat space, one obtains the five-dimensional fuzzball solutions with momentum that were constructed in [16 – 19]. These solutions were originally obtained by applying a spectral flow transformation to the five-dimensional solutions without momentum (3.15). They carry the following 5D charges

$$\begin{aligned} J^3 &= -\frac{N_1 N_5}{2}\left(2m + \frac{1}{n}\right), & \bar{J}^3 &= -\frac{N_1 N_5}{2n}, \\ P &= N_1 N_5 m\left(m + \frac{1}{n}\right), \end{aligned} \tag{3.19}$$

where P denotes the momentum on the S^4 circle. The flux quantization discussed in paragraph 2.3.2 imposes that the parameter m should be an integer.

4. Microscopic interpretation

We will now discuss the microscopic interpretation of the solutions we considered both from the 4D and 5D point of view. Let us start with the configurations (2.34) without

D0-charge in frame A. We showed that these arise, through the 4D-5D connection, from 5D fuzzball solutions with circular profile which carry macroscopic angular momentum $J_{12} = N_1 N_5/n$ and are placed in a Taub-NUT geometry. A first question is whether we should regard these solutions as zero-entropy constituents of a spinning black hole or of a black ring in five dimensions. In the present context, the latter is the only possibility, since a black hole of the desired charge (if it exists as a BPS solution in type II on a torus) cannot be placed in Taub-NUT space in a supersymmetric manner and therefore the 4D-5D connection cannot be applied to it. Indeed, if it could, the resulting 4D configuration would be a small black hole with charges $(0, N_K, N_1, N_5, N_1N_5/n, 0, 0, 0)$. This is however a polar charge for which there cannot exist a single center black hole solution, even including higher derivative corrections. Hence we should see our 4D solutions as coming from small black ring microstates in five dimensions. This interpretation also corresponds to the one argued in [14, 52 – 54]. We want to point out that the above argument does not rule out the existence of a 5D supersymmetric spinless ($J_{12} = J_{34} = 0$) small black hole placed at the center of Taub-NUT space. Indeed, the resulting 4D configuration would have pure D4-charge $(0, N_K, N_1, N_5, 0, 0, 0, 0)$, which is not a polar charge ($\hat{q}_0 = 0$), and therefore could give rise to a single-centered small black hole when higher derivative corrections are taken into account.

Let us review which states in the dual CFT correspond to the configurations (2.34) from the 5D point of view. The D1-D5 CFT is a deformation of a symmetric product CFT with target space $(T_2 \times T_3)^{N_1N_5}/S_{N_1N_5}$ (see [55] for a review). For our purposes, we can consider the theory at the orbifold point. The states we are considering are closely related to chiral primary operators denoted by σ_n^{--} with quantum numbers $L_0 = J^3 = \bar{L}_0 = \bar{J}^3 = \frac{n-1}{2}$. We can construct operators $\mathrm{U}(\alpha)$ which generate a left-moving spectral flow with an integer parameter α:

$$\begin{aligned} \mathrm{U}(\alpha) L_0 \, \mathrm{U}(\alpha)^{-1} &= L_0 - \alpha J^3 + \alpha^2 \frac{c}{24} \\ \mathrm{U}(\alpha) J^3 \, \mathrm{U}(\alpha)^{-1} &= J^3 - \alpha \frac{c}{12} \end{aligned} \tag{4.1}$$

where the central charge is $c = 6N_1N_5$. Similar generators of right-moving spectral flow with parameter $\tilde{\alpha}$ will be denoted by $\tilde{U}(\tilde{\alpha})$. The CFT states corresponding to (2.34) are ground states in the R sector given by

$$\mathrm{U}(1)\tilde{\mathrm{U}}(1)(\sigma_n^{--})^{\frac{N_1N_5}{n}}|0\rangle. \tag{4.2}$$

They carry the quantum numbers

$$\begin{aligned} L_0 &= \frac{N_1N_5}{4}\,, & \bar{L}_0 &= \frac{N_1N_5}{4}\,, \\ J^3 &= -\frac{N_1N_5}{2n}\,, & \bar{J}^3 &= -\frac{N_1N_5}{2n}\,, \end{aligned} \tag{4.3}$$

$$P = L_0 - \bar{L}_0 = 0\,. \tag{4.4}$$

The above states belong to a 'microcanonical' ensemble of R ground states at fixed D1-charge N_1, D5-charge N_5, and angular momenta[8] $J_{12} = N_1N_5/n,\ J_{34} = 0$. When $n \gg 1$, J_{12} is sufficiently far from the maximal value N_1N_5, and there is an exponential degeneracy of states carrying these quantum numbers, leading to a microscopic entropy [54]

$$S_{\rm micro} = 2\sqrt{2}\pi\sqrt{N_1N_5 - J} = 2\sqrt{2}\pi\sqrt{N_1N_5\left(1 - \frac{1}{n}\right)}\,. \tag{4.5}$$

It is expected on the basis of general arguments [56] that, after including higher derivative corrections to the effective action, there exists a black ring solution with a matching macroscopic entropy. It is an open problem to explicitly compute such corrections in toroidal compactifications, unlike the case where the four-torus $T_2 \times T_3$ is replaced with K_3 [52, 53, 57].

When a small black ring is placed in Taub-NUT space with one unit of NUT charge and the radius of the Taub-NUT circle is decreased, one obtains a 4D configuration consisting of two centers. One center, coming from the wrapped ring itself, becomes a small black hole in 4D, while the other center, coming from the Taub-NUT charge, is a KK monopole carrying zero entropy [52, 53]. In our duality frame A, the first center is a small $D4-D2$ black hole with charge $(0, 0, N_1, N_5, N_1N_5/n, 0, 0, 0)$ and entropy given by (4.5) and the second center is a pure D4-brane with charge $(0, 1, 0, 0, 0, 0, 0, 0)$. Because these charges are not parallel, the combined system carries macroscopic angular momentum $J_z = -N_1N_5/n$. Therefore we can see our 4D polar D6-anti D6 configurations (2.34) as zero-entropy constituents of this two-centered configuration.

A similar discussion can be made for the solutions (2.47) carrying D0-charge in frame A. Their CFT counterparts are related to (4.2) by an additional left-moving spectral flow with parameter $2m$:

$$\mathrm{U}(2m+1)\tilde{\mathrm{U}}(1)(\sigma_n^{--})^{\frac{N_1N_5}{n}}|0\rangle. \tag{4.6}$$

They carry the quantum numbers that were anticipated in (1.1):

$$\begin{aligned}
L_0 &= N_1N_5\left(m^2 + \frac{m}{n} + 1/4\right)\,, & \bar{L}_0 &= \frac{N_1N_5}{4}\,,\\
J^3 &= -\frac{N_1N_5}{2}\left(2m + \frac{1}{n}\right)\,, & \bar{J}^3 &= -\frac{N_1N_5}{2n}\,,\\
P &= L_0 - \bar{L}_0 = N_1N_5m\left(m + \frac{1}{n}\right)\,.
\end{aligned} \tag{4.7}$$

In the CFT, the parameters n and m should be quantized such that n is a divisor of N_1N_5 and m is an integer. This matches with the conditions we found from charge quantization in the corresponding D-brane configurations. These states are part of an ensemble of CFT states with fixed D1-D5 charges, angular momenta $J^3, \bar{J}^3$ and momentum P. This ensemble is obtained by the ensemble of zero momentum ground states discussed above by acting with the spectral flow operator $\mathrm{U}(2m)$. The degeneracy is then again given by (4.5).

[8] A different ensemble, where the angular momenta are not fixed, was advocated in the light of the OSV conjecture in [53]

5. Discussion

In this paper we have identified four-dimensional multicenter D-brane configurations that correspond to a class of fuzzball solutions in five noncompact dimensions under the 4D-5D connection. In a type IIA duality frame where all the charges come from D6-D4-D2-D0 branes, the relevant 4D configurations are two-centered D6-anti D6 solutions with fluxes corresponding to polar states .

The fuzzball solutions considered here were highly symmetric, where the profile function that defines the solution is taken to be a circular curve in the $x^1 - x^2$ plane in the coordinates (3.16). Let us first comment on the fate of more general fuzzball solutions under the 4D-5D connection. A fuzzball solution arising from a generic curve will typically not have enough symmetry to be written as a torus fibration over a four-dimensional base as in (3.2) and can hence not be given a four-dimensional interpretation. However, according to the proposed dictionary between microstates and fuzzball solutions in [58, 59], the subclass of fuzzball solutions that semiclassically represent eigenstates of the R-symmetry group should possess $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry and be represented by (possibly disconnected) circular curves in the $x^1 - x^2$ and $x^3 - x^4$ planes in the coordinates (3.2). Such solutions have isometries along the directions $\partial/\partial\phi$ and $\partial/\partial\psi$ as well as along the Taub-Nut direction $\partial/\partial x^4$, and should therefore be the lift of axially symmetric solutions in four dimensions. When the quantum numbers are chosen appropriately, these would describe other constituents of the 4-dimensional 2-centered system with entropy (4.5). It would be interesting to explore this ensemble of four-dimensional configurations.

We would also like to comment on the relation between the present work and black hole deconstruction [7]. In four dimensions, say in our frame A, there exist multicentered 'scaling' solutions with centers so close that their throats have 'melted' together and which are asymptotically indistinguishable from single centered solutions. Such solutions can carry the same charges as a large single-centered D4-D0 black hole, and can be seen as a deconstruction of such a black hole into zero-entropy constitutents. The scaling solutions consist of a 'core' D6 anti-D6 system with flux, and a 'halo'of D0-brane centers added to it (again, see [1] for more details on the formalism of 'cores' and 'halos'). The scaling limit consists of taking the total D0-charge to be parametrically larger than the magnetic charge $p^1 p^2 p^3$. The entropy of the black hole in this limit can be understood by treating the D0-branes as probes and counting the supersymmetric ground states of the probe quantum mechanics [60]. The 'core' D6 anti-D6 system in these configurations is precisely of the kind that we studied in this paper and mapped to 5D fuzzball solutions. Indeed, for the special values $n = 1,\ m = -1/2$ of our parameters we obtain the following charges at the centers

$$\begin{aligned}
\Gamma_1 &= \left(-1, \frac{N_K}{2}, \frac{N_1}{2}, \frac{N_5}{2}, -\frac{N_1 N_5}{4}, -\frac{N_K N_5}{4}, -\frac{N_K N_1}{4}, \frac{N_K N_1 N_5}{8}\right), \\
\Gamma_2 &= \left(1, \frac{N_K}{2}, \frac{N_1}{2}, \frac{N_5}{2}, \frac{N_1 N_5}{4}, \frac{N_K N_5}{4}, \frac{N_K N_1}{4}, \frac{N_K N_1 N_5}{8}\right).
\end{aligned} \tag{5.1}$$

These are precisely the charges that appear in the core of the scaling solutions in [7]. It seems natural to expect that, for the other values of our parameters m and n, our

q_0	$P(S^4)$	p^0	$KKmon(S^4)$
q_1	$P(S^5)$	p^1	$KKmon(S^5)$
$\tilde{q}_2$	$D1(S^4)$	$\tilde{p}^2$	$D5(S^4)$
q_3	$D1(S^5)$	p^3	$D5(S^5)$

Table 4: The interpretation of the charge in an intermediate frame $\tilde{B}$.

configurations can serve as the core system for the deconstruction of a black hole with added D2-charge.

The relation to deconstruction could have interesting implications in five dimensions as well. If we take a scaling solution in four dimensions, dualize it to frame B and take the 4D-5D limit, we should end up with a configuration carrying the charges of a large D1-D5-P Strominger-Vafa [61] black hole. The scaling limit implies that we will have $P \gg N_1 N_5$, which is equivalent to the Cardy limit $\Lambda_0 \ll c$ where the CFT microstate counting is performed. Therefore such configurations would be candidates for describing typical microstates of the D1-D5-P black hole, and it would be interesting to study such solutions in more detail. It is not clear whether such configurations could rightly be called 'fuzzball' geometries for the D1-D5-P black hole, as they will not be smooth near the centers where the harmonic functions describing the momentum diverge. As argued in [62], treating the momentum as coming from giant graviton probes, the number of ground states would be of the right order to explain the entropy.

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A. Reduction formulas in frame B

We now discuss the dimensional reduction of type II on T^6 in the duality frame B to the bosonic STU model action (2.1). The 10-dimensional interpretation of the U(1) charges is given in table 3. It will be convenient to first reduce to an intermediate duality frame, which we will call frame $\tilde{B}$, where the U(1) fields are labeled as $\mathcal{A}^0, \mathcal{A}^1, \mathcal{B}_2, \mathcal{A}^3$ and the charges are labeled as $(p^0, p^1, \tilde{p}^2, p^3, q_1, \tilde{q}_2, q_1, q_0)$. The 10D interpretation of the charges in frame $\tilde{B}$ is given in table 4. The frame $\tilde{B}$ differs from the frame B of table 3 by an electromagnetic duality transformation on the U(1) field $\mathcal{B}^2$.

It suffices to restrict attention to a truncated IIB action containing only the metric, dilaton and RR 3-form:

$$S = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \sqrt{-G^{(10)}} \left[e^{-2\Phi^{(10)}} \left(R^{(10)} + 4\partial_M \Phi^{(10)} \partial^M \Phi^{(10)} \right) - \frac{1}{12} F^{(10)}_{MNP} F^{(10)\ MNP} \right]. \tag{A.1}$$

We perform a trivial dimensional reduction over the four-torus $T_2 \times T_3$, while allowing the torus T_1 to be nontrivially fibered over the four-dimensional base. We start by flipping the sign of $\Phi^{(10)}$ and making a Weyl transformation (as one does in S-duality) such that all terms in (A.1) have an $e^{-2\Phi^{(10)}}$ factor in front. We can then perform the dimensional reduction of this sector as discussed in [63]. We will here follow closely the conventions of [64]. We take the following reduction ansatz

$$\begin{aligned}
\Phi^{(10)} &= -\Phi - \frac{1}{4} \ln \det \hat{G}_{mn} - \frac{1}{4} \ln \det \hat{G}_{ij} \,, \\
G^{(10)}_{\mu\nu} &= (\det \hat{G})^{-1/4} \left(e^{\Phi} G_{\mu\nu} + 2\beta^2 e^{-\Phi} \mathcal{A}^{m-4}_{\mu} \mathcal{A}^{n-4}_{\nu} \hat{G}_{mn} \right) , \\
G^{(10)}_{\mu n} &= \sqrt{2}\beta (\det \hat{G})^{-1/4} e^{-\Phi} \hat{G}_{np} \mathcal{A}^{p-4}_{\mu} \,, \\
G^{(10)}_{mn} &= (\det \hat{G})^{-1/4} e^{-\Phi} \hat{G}_{mn} \,, \\
G^{(10)}_{ij} &= (\det \hat{G})^{-1/4} e^{-\Phi} \hat{G}_{ij} \,, \\
C^{(10)}_{\mu\nu} &= C_{\mu\nu} + 2\beta^2 \hat{C}_{45} (\mathcal{A}^0_{\mu} \mathcal{A}^1_{\nu} - \mathcal{A}^1_{\mu} \mathcal{A}^0_{\nu}) + \beta^2 (\mathcal{A}^0_{\mu} \mathcal{B}_{2\nu} - \mathcal{B}_{2\mu} \mathcal{A}^0_{\nu}) + \beta^2 (\mathcal{A}^1_{\mu} \mathcal{A}^3_{\nu} - \mathcal{A}^3_{\mu} \mathcal{A}^1_{\nu}) \,, \\
C^{(10)}_{\mu 4} &= \sqrt{2}\beta (\mathcal{B}_{2\mu} + \hat{C}_{45} \mathcal{A}^1_{\mu}) \,, \\
C^{(10)}_{\mu 5} &= \sqrt{2}\beta^2 (\mathcal{A}^3_{\mu} - \hat{C}_{45} \mathcal{A}^0_{\mu}) \,, \\
C^{(10)}_{mn} &= \hat{C}_{mn} \,.
\end{aligned} \tag{A.2}$$

Here, $M, N = 0, \ldots, 9$; $m, n = 4, 5$, $i, j = 6, \ldots 9$ and we have taken x^4, x^5 to parametrize S_4, S_5 respectively.

The matrix $\hat{G}_{ij}$ is a constant metric on $T_2 \times T_3$ and the matrices $\hat{G}_{mn}, \hat{C}_{mn}$ can be conveniently parametrized as

$$\begin{aligned}
\hat{G}_{mn} &= b_3 \begin{pmatrix} \frac{a_1^2 + b_1^2}{b_1} & -\frac{a_1}{b_1} \\ -\frac{a_1}{b_1} & \frac{1}{b_1} \end{pmatrix} , \\
\hat{C}_{mn} &= \begin{pmatrix} 0 & a_3 \\ -a_3 & 0 \end{pmatrix} , \\
e^{-2\Phi} &= b_2 \,.
\end{aligned} \tag{A.3}$$

The two-form $C_{\mu\nu}$ can be dualized in four dimensions to give another scalar $\tilde{a}_1$:

$$da_2 = b_2^2 \star F \,, \tag{A.4}$$

where the Hodge $\star$ is to be taken with respect to the 4D metric $G_{\mu\nu}$ and the three-form field strength F is defined as

$$F = dC + \frac{\beta^2}{2}\left(\mathcal{A}^0 \wedge \mathcal{G}^2 + \mathcal{B}^2 \wedge \mathcal{F}^0 + \mathcal{A}^1 \wedge \mathcal{F}^3 + \mathcal{A}^3 \wedge \mathcal{F}^1\right) . \tag{A.5}$$

From the above expressions it is clear that $z^1 = a_1 + ib_1$ is the complex structure modulus of T_1, $z^2 = a_2 + ib_2$ is the 4D axion-dilaton and $z^3 = a_3 + ib_3$ is the complexified Kähler modulus of T_1.

In these variables, one finds after performing the dimensional reduction the 4D action

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-G}\Big[R - 2\sum_{A=1}^{3}\frac{\partial_\mu \tilde{z}^A \partial^\mu \bar{\tilde{z}}^A}{(\tilde{z}^A - \bar{\tilde{z}}^A)^2} + \frac{\beta^2}{2}\mathrm{Im}\tilde{\mathcal{N}}_{IJ}\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J\,\mu\nu} + \frac{\beta^2}{4}\mathrm{Re}\tilde{\mathcal{N}}_{IJ}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma}\Big], . \tag{A.6}$$

with the matrix $\tilde{\mathcal{N}}$ given by

$$\mathrm{Re}\tilde{\mathcal{N}} = \begin{pmatrix} 0 & 0 & -a_2 & 0 \\ 0 & 0 & 0 & -a_2 \\ -a_2 & 0 & 0 & 0 \\ 0 & -a_2 & 0 & 0 \end{pmatrix},$$

$$\mathrm{Im}\tilde{\mathcal{N}} = \begin{pmatrix} -\frac{b_2\left(a_1{}^2+b_1{}^2\right)\left(a_3{}^2+b_3{}^2\right)}{b_1 b_3} & \frac{a_1 b_2\left(a_3{}^2+b_3{}^2\right)}{b_1 b_3} & -\frac{a_1 a_3 b_2}{b_1 b_3} & \frac{a_3 b_2\left(a_1{}^2+b_1{}^2\right)}{b_1 b_3} \\ \frac{a_1 b_2\left(a_3{}^2+b_3{}^2\right)}{b_1 b_3} & -\frac{b_2\left(a_3{}^2+b_3{}^2\right)}{b_1 b_3} & \frac{a_3 b_2}{b_1 b_3} & -\frac{a_1 a_3 b_2}{b_1 b_3} \\ -\frac{a_1 a_3 b_2}{b_1 b_3} & \frac{a_3 b_2}{b_1 b_3} & -\frac{b_2}{b_1 b_3} & \frac{a_1 b_2}{b_1 b_3} \\ \frac{a_3 b_2\left(a_1{}^2+b_1{}^2\right)}{b_1 b_3} & -\frac{a_1 a_3 b_2}{b_1 b_3} & \frac{a_1 b_2}{b_1 b_3} & -\frac{b_2\left(a_1{}^2+b_1{}^2\right)}{b_1 b_3} \end{pmatrix}.$$

The 4-dimensional Newton constant G_4 is given by

$$G_4 = \frac{8\pi^6(\alpha')^4 g^2}{(2\pi)^2 R_4 R_5 V_{T_2\times T_3}}, \tag{A.7}$$

with g the string coupling in 10 dimensions.

To go to the duality frame B of table 3, where the U(1) fields are labeled as $\mathcal{A}^0, \mathcal{A}^1, \mathcal{B}^2, \mathcal{A}^3$ and then charges are labeled as $(p^0, p^1, p^2, p^3, q_1, q_2, q_3, q_0)$, we have to perform an electromagnetic duality on the field $\mathcal{B}_2$. After this duality, the action takes the form (2.1) with the matrix $\mathcal{N}$ related to $\tilde{\mathcal{N}}$ given above by a symplectic transformation

$$\mathcal{N} = (C + D\tilde{\mathcal{N}})(A + B\tilde{\mathcal{N}})^{-1}, \tag{A.8}$$

with

$$A = D = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; B = -C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.9}$$

Explicitly, one finds

$$\mathrm{Re}\mathcal{N} = -\begin{pmatrix} 2\,a_1\,a_2\,a_3 & -(a_2\,a_3) & -(a_1\,a_3) & -(a_1\,a_2) \\ -(a_2\,a_3) & 0 & a_3 & a_2 \\ -(a_1\,a_3) & a_3 & 0 & a_1 \\ -(a_1\,a_2) & a_2 & a_1 & 0 \end{pmatrix},$$

$$\mathrm{Im}\mathcal{N} = -\begin{pmatrix} b_1b_2b_3+\frac{b_1b_2a_3^2}{b_3}+\frac{b_1b_3a_2^2}{b_2}+\frac{b_2b_3a_1^2}{b_1} & -\frac{a_1\,b_2\,b_3}{b_1} & -\frac{a_2\,b_1\,b_3}{b_2} & -\frac{a_3\,b_1\,b_2}{b_3} \\ -\frac{a_1\,b_2\,b_3}{b_1} & \frac{b_2\,b_3}{b_1} & 0 & 0 \\ -\frac{a_2\,b_1\,b_3}{b_2} & 0 & \frac{b_1\,b_3}{b_2} & 0 \\ -\frac{a_3\,b_1\,b_2}{b_3} & 0 & 0 & \frac{b_1\,b_2}{b_3} \end{pmatrix}.$$

This is indeed the standard form of the matrix $\mathcal{N}$ in the STU-model derived from the prepotential through (2.2). The U(1) field $\mathcal{B}_2$ is related to the $\mathcal{A}^I$ through

$$d\mathcal{B}_2 = \mathrm{Im}\mathcal{N}_{2J} \star \mathcal{F}^J + \mathrm{Re}\mathcal{N}_{2J}\mathcal{F}^J\,. \tag{A.10}$$

Summarized, we have found the following reduction formulas

$$\begin{aligned} e^{2\Phi^{(10)}} &= \frac{b^2}{b^3}\,, \\ ds_{10}^2 &= \frac{1}{\sqrt{b^2b^3}}ds_4^2 + \sqrt{b^2b^3}\mathcal{M}_{mn}(dx^m+\sqrt{2}\beta\mathcal{A}^{m-4})(dx^n+\sqrt{2}\beta\mathcal{A}^{n-4}) + \sqrt{\frac{b^2}{b^3}}ds^2_{T_2\times T_3}\,, \\ \mathcal{M}_{mn} &= \frac{1}{b^1}\begin{pmatrix} (a^1)^2+(b^1)^2 & -a^1 \\ -a^1 & 1 \end{pmatrix}, \\ C^{(10)} &= \frac{1}{2}C_{\mu\nu}dx^\mu dx^\nu + a^3(dx^4-\sqrt{2}\beta\mathcal{A}^0)\wedge(dx^5-\sqrt{2}\beta\mathcal{A}^1) \\ &\qquad -\sqrt{2}\beta dx^4\wedge\mathcal{B}^2 - \sqrt{2}\beta dx^5\wedge\mathcal{A}^3 + \beta^2\left(\mathcal{A}^0\wedge\mathcal{B}^2+\mathcal{A}^1\wedge\mathcal{A}^3\right), \\ da_2 &= (b_2)^2\star F\,, \\ F &= dC + \frac{\beta^2}{2}(\mathcal{A}^0\wedge\mathcal{G}^2 + \mathcal{B}^2\wedge\mathcal{F}^0 + \mathcal{A}^1\wedge\mathcal{F}^3 + \mathcal{A}^3\wedge\mathcal{F}^1)\,. \end{aligned} \tag{A.11}$$

References

[1] F. Denef and G.W. Moore, *Split states, entropy enigmas, holes and halos*, hep-th/0702146.

[2] F. Denef, *Supergravity flows and D-brane stability*, *JHEP* **08** (2000) 050 [hep-th/0005049].

[3] F. Denef, B.R. Greene and M. Raugas, *Split attractor flows and the spectrum of BPS D-branes on the quintic*, *JHEP* **05** (2001) 012 [hep-th/0101135].

[4] B. Bates and F. Denef, *Exact solutions for supersymmetric stationary black hole composites*, hep-th/0304094.

[5] J. de Boer, F. Denef, S. El-Showk, I. Messamah and D. Van den Bleeken, *Black hole bound states in $AdS_3\times S^2$*, arXiv:0802.2257.

[6] H. Ooguri, A. Strominger and C. Vafa, *Black hole attractors and the topological string*, *Phys. Rev.* **D 70** (2004) 106007 [hep-th/0405146].

[7] F. Denef, D. Gaiotto, A. Strominger, D. Van den Bleeken and X. Yin, *Black hole deconstruction*, hep-th/0703252.

JHEP10(2008)039

[8] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *A supersymmetric black ring*, *Phys. Rev. Lett.* **93** (2004) 211302 [hep-th/0407065].

[9] I. Bena and N.P. Warner, *One ring to rule them all... and in the darkness bind them?*, *Adv. Theor. Math. Phys.* **9** (2005) 667 [hep-th/0408106].

[10] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric black rings and three-charge supertubes*, *Phys. Rev.* **D 71** (2005) 024033 [hep-th/0408120].

[11] R. Emparan and H.S. Reall, *Black rings*, *Class. and Quant. Grav.* **23** (2006) R169 [hep-th/0608012].

[12] J.M. Maldacena and L. Maoz, *De-singularization by rotation*, *JHEP* **12** (2002) 055 [hep-th/0012025].

[13] O. Lunin and S.D. Mathur, *AdS/CFT duality and the black hole information paradox*, *Nucl. Phys.* **B 623** (2002) 342 [hep-th/0109154].

[14] O. Lunin and S.D. Mathur, *Statistical interpretation of Bekenstein entropy for systems with a stretched horizon*, *Phys. Rev. Lett.* **88** (2002) 211303 [hep-th/0202072].

[15] O. Lunin, J.M. Maldacena and L. Maoz, *Gravity solutions for the $D1$-$D5$ system with angular momentum*, hep-th/0212210.

[16] O. Lunin, *Adding momentum to $D1$-$D5$ system*, *JHEP* **04** (2004) 054 [hep-th/0404006].

[17] S. Giusto, S.D. Mathur and A. Saxena, *3-charge geometries and their CFT duals*, *Nucl. Phys.* **B 710** (2005) 425 [hep-th/0406103].

[18] S. Giusto, S.D. Mathur and A. Saxena, *Dual geometries for a set of 3-charge microstates*, *Nucl. Phys.* **B 701** (2004) 357 [hep-th/0405017].

[19] S. Giusto and S.D. Mathur, *Geometry of $D1$-$D5$-P bound states*, *Nucl. Phys.* **B 729** (2005) 203 [hep-th/0409067].

[20] I. Bena and P. Kraus, *Microstates of the $D1$-$D5$-KK system*, *Phys. Rev.* **D 72** (2005) 025007 [hep-th/0503053].

[21] M. Taylor, *General 2 charge geometries*, *JHEP* **03** (2006) 009 [hep-th/0507223].

[22] A. Saxena, G. Potvin, S. Giusto and A.W. Peet, *Smooth geometries with four charges in four dimensions*, *JHEP* **04** (2006) 010 [hep-th/0509214].

[23] S. Giusto, S.D. Mathur and Y.K. Srivastava, *Dynamics of supertubes*, *Nucl. Phys.* **B 754** (2006) 233 [hep-th/0510235].

[24] S. Giusto, S.D. Mathur and Y.K. Srivastava, *A microstate for the 3-charge black ring*, *Nucl. Phys.* **B 763** (2007) 60 [hep-th/0601193].

[25] I. Kanitscheider, K. Skenderis and M. Taylor, *Fuzzballs with internal excitations*, *JHEP* **06** (2007) 056 [arXiv:0704.0690].

[26] S.D. Mathur, *The fuzzball proposal for black holes: an elementary review*, *Fortschr. Phys.* **53** (2005) 793 [hep-th/0502050].

[27] K. Skenderis and M. Taylor, *The fuzzball proposal for black holes*, *Phys. Rept.* **467** (2008) 117 [arXiv:0804.0552].

[28] D. Gaiotto, A. Strominger and X. Yin, *New connections between $4D$ and $5D$ black holes*, *JHEP* **02** (2006) 024 [hep-th/0503217].

JHEP10(2008)039

[29] D. Gaiotto, A. Strominger and X. Yin, *5D black rings and 4D black holes*, *JHEP* **02** (2006) 023 [hep-th/0504126].

[30] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric 4D rotating black holes from 5D black rings*, *JHEP* **08** (2005) 042 [hep-th/0504125].

[31] I. Bena, P. Kraus and N.P. Warner, *Black rings in Taub-NUT*, *Phys. Rev.* **D 72** (2005) 084019 [hep-th/0504142].

[32] K. Behrndt, G. Lopes Cardoso and S. Mahapatra, *Exploring the relation between 4D and 5D BPS solutions*, *Nucl. Phys.* **B 732** (2006) 200 [hep-th/0506251].

[33] J. Ford, S. Giusto, A. Peet and A. Saxena, *Reduction without reduction: adding KK-monopoles to five dimensional stationary axisymmetric solutions*, *Class. and Quant. Grav.* **25** (2008) 075014 [arXiv:0708.3823].

[34] I. Bena and N.P. Warner, *Bubbling supertubes and foaming black holes*, *Phys. Rev.* **D 74** (2006) 066001 [hep-th/0505166].

[35] P. Berglund, E.G. Gimon and T.S. Levi, *Supergravity microstates for BPS black holes and black rings*, *JHEP* **06** (2006) 007 [hep-th/0505167].

[36] I. Bena, C.-W. Wang and N.P. Warner, *The foaming three-charge black hole*, *Phys. Rev.* **D 75** (2007) 124026 [hep-th/0604110].

[37] V. Balasubramanian, E.G. Gimon and T.S. Levi, *Four dimensional black hole microstates: from D-branes to spacetime foam*, *JHEP* **01** (2008) 056 [hep-th/0606118].

[38] I. Bena, C.-W. Wang and N.P. Warner, *Mergers and typical black hole microstates*, *JHEP* **11** (2006) 042 [hep-th/0608217].

[39] M.C.N. Cheng, *More bubbling solutions*, *JHEP* **03** (2007) 070 [hep-th/0611156].

[40] I. Bena, N. Bobev and N.P. Warner, *Bubbles on manifolds with a* U(1) *isometry*, *JHEP* **08** (2007) 004 [arXiv:0705.3641].

[41] E.G. Gimon and T.S. Levi, *Black ring deconstruction*, *JHEP* **04** (2008) 098 [arXiv:0706.3394].

[42] I. Bena, C.-W. Wang and N.P. Warner, *Plumbing the abyss: black ring microstates*, *JHEP* **07** (2008) 019 [arXiv:0706.3786].

[43] I. Bena and N.P. Warner, *Black holes, black rings and their microstates*, hep-th/0701216.

[44] M.J. Duff, J.T. Liu and J. Rahmfeld, *Four-dimensional string-string-string triality*, *Nucl. Phys.* **B 459** (1996) 125 [hep-th/9508094].

[45] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W.K. Wong, *STU black holes and string triality*, *Phys. Rev.* **D 54** (1996) 6293 [hep-th/9608059].

[46] M. Billó et al., *The 0-brane action in a general $D = 4$ supergravity background*, *Class. and Quant. Grav.* **16** (1999) 2335 [hep-th/9902100].

[47] J. de Boer, M.C.N. Cheng, R. Dijkgraaf, J. Manschot and E. Verlinde, *A Farey tail for attractor black holes*, *JHEP* **11** (2006) 024 [hep-th/0608059].

[48] I. Bena, N. Bobev and N.P. Warner, *Spectral flow and the spectrum of multi-center solutions*, *Phys. Rev.* **D 77** (2008) 125025 [arXiv:0803.1203].

[49] G.W. Moore, *Arithmetic and attractors*, hep-th/9807087.

JHEP10(2008)039

[50] F. Denef, *Quantum quivers and Hall/hole halos*, *JHEP* **10** (2002) 023 [hep-th/0206072].

[51] J.B. Gutowski, D. Martelli and H.S. Reall, *All supersymmetric solutions of minimal supergravity in six dimensions*, *Class. and Quant. Grav.* **20** (2003) 5049 [hep-th/0306235].

[52] N. Iizuka and M. Shigemori, *A note on $D1$-$D5$-J system and $5D$ small black ring*, *JHEP* **08** (2005) 100 [hep-th/0506215].

[53] A. Dabholkar, N. Iizuka, A. Iqubal and M. Shigemori, *Precision microstate counting of small black rings*, *Phys. Rev. Lett.* **96** (2006) 071601 [hep-th/0511120].

[54] V. Balasubramanian, P. Kraus and M. Shigemori, *Massless black holes and black rings as effective geometries of the $D1$-$D5$ system*, *Class. and Quant. Grav.* **22** (2005) 4803 [hep-th/0508110].

[55] J.R. David, G. Mandal and S.R. Wadia, *Microscopic formulation of black holes in string theory*, *Phys. Rept.* **369** (2002) 549 [hep-th/0203048].

[56] A. Sen, *Extremal black holes and elementary string states*, *Mod. Phys. Lett.* **A 10** (1995) 2081 [hep-th/9504147].

[57] A. Dabholkar, N. Iizuka, A. Iqubal, A. Sen and M. Shigemori, *Spinning strings as small black rings*, *JHEP* **04** (2007) 017 [hep-th/0611166].

[58] K. Skenderis and M. Taylor, *Fuzzball solutions and $D1$-$D5$ microstates*, *Phys. Rev. Lett.* **98** (2007) 071601 [hep-th/0609154].

[59] I. Kanitscheider, K. Skenderis and M. Taylor, *Holographic anatomy of fuzzballs*, *JHEP* **04** (2007) 023 [hep-th/0611171].

[60] D. Gaiotto, A. Strominger and X. Yin, *Superconformal black hole quantum mechanics*, *JHEP* **11** (2005) 017 [hep-th/0412322].

[61] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029].

[62] J. Raeymaekers, *Near-horizon microstates of the $D1$-$D5$-P black hole*, *JHEP* **02** (2008) 006 [arXiv:0710.4912].

[63] J. Maharana and J.H. Schwarz, *Noncompact symmetries in string theory*, *Nucl. Phys.* **B 390** (1993) 3 [hep-th/9207016].

[64] A. Sen, *Strong-weak coupling duality in four-dimensional string theory*, *Int. J. Mod. Phys.* **A 9** (1994) 3707 [hep-th/9402002].

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